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On the skewness of the temperature derivative in turbulent flows

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This note provides some explanation of the fact that, contrary to the requirements of local isotropy, the skewness S of the streamwise temperature derivative $\partial\theta/\partial x_1$ has been observed to be a non-zero constant of magnitude of about unity in high-Reynolds-number and high-Péclet-number turbulent shear flows. Measurements in slightly heated homogeneous shear flows and in unsheared grid turbulence suggest that S is non-zero only when the mean shear dU_1/dx_2 and the mean temperature gradient dT/dx_2 are both non-zero. The sign of S is given by $-\operatorname{sgn}(dU_1/dx_2).\operatorname{sgn}(dT/dx_2)$. For fixed dU_1/dx_2, S is of the form $\tanh(\alpha\, dT/dx_2)$, α being a constant, while for fixed dT/dx_2, it is of the form $S/S^* = 1-\beta_1 \exp(-\beta_2\tau)$, where S^* is a characteristic value of S, β_1 and β_2 are positive constants, and τ can be interpreted as a 'total strain'. The derivative skewness data in other (inhomogeneous) shear flows are also compatible with the latter relation. Predictions from a simplified transport equation for $\overline{(\partial\theta/\partial x_1)^3}$, derived in the light of the present experimental observations, are in reasonable agreement with the measured values of S. A possible physical mechanism maintaining S is discussed.

1. Introduction

The concept of local isotropy, introduced by Kolmogorov (1941), implies that the small-scale properties in high-Reynolds-number turbulence are isotropic irrespective of the gross details of the flow. Oboukhov (1949) and Corrsin (1952) independently suggested that local isotropy must be applicable to transported scalar fields as well, provided that the Reynolds and Péclet numbers are sufficiently large. If locally isotropic effects indeed dominate small scale properties such as the spatial derivatives of the temperature θ, invariance by reflexion about the x_1 axis (the direction of the mean flow speed U_1) requires that the derivative skewness $S \equiv \overline{(\partial\theta/\partial x_1)^3}/\overline{(\partial\theta/\partial x_1)^2}^{\frac{3}{2}}$ should vanish. However, measurements in shear flows with heat transfer have shown that S is of order unity for a relatively wide range of Reynolds numbers (table 1). As shown by Gibson, Friehe & McConnell (1977) and Mestayer *et al.* (1976), the sign of this parameter is determined by the signs of the mean velocity and the mean temperature gradients.

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Flow	Source	R_λ	$\|S\|$	Average $\|S\|$
Heated wake	Freymuth & Uberoi† (1971)	126	0·6	
		188	0·6	
	Gibson *et al.* (1977)	336	0·81	
Cooled wake	Gibson *et al.* (1977)	336	1·08	
Heated jet	Antonia & Van Atta (1975)	240	0·70	
	Gibson *et al.* (1977)	457	0·72	0·83
		1154	0·87	
	Sreenivasan & Antonia (1977)	100	0·99	
		185	0·75	
		240	0·98	
		255	1·00	
Heated boundary layer	Gibson, Stegen & Williams (1970)	400	0·86	
		540	0·65	
		750	0·89	
		950	0·56	
	Mestayer *et al.* (1976)	890	0·96	
		920	0·77	
		975	0·85	
		1100	0·77	
		1140	0·77	
		1200	0·71	0·82
		1200	0·79	
	Gibson *et al.* (1977)	1095	0·85	
		1126	0·75	
	Sreenivasan *et al.* (1977)	135	0·99	
		155	1·00	
		165	0·98	
		180	1·00	
	Sreenivasan & Antonia (1977)	1100	0·67	

† Only the data for $R_\lambda \gtrsim 100$ are given here. The microscale Reynolds number $R_\lambda \equiv u_1' \lambda/\nu$.

TABLE 1. Collection of data on the skewness of the temperature derivative in inhomogeneous shear flows.

The non-zero value of S implies a ramp-like character for the temperature signal; in fact, Antonia, Prabhu & Stephenson (1975), Mestayer *et al.* (1976) and Gibson *et al.* (1977) have observed that the temperature signal in shear flows possesses a large-scale ramp-like feature with a sharp leading (or trailing) edge and a gradually tapering trailing (or leading) edge. Furthermore, Sreenivasan, Antonia & Britz (1979) showed that these *large-scale* ramps contribute most of the observed magnitude of the skewness. Gibson *et al.* (1977) and Mestayer *et al.* (1976) also suggested that the presence of the mean shear is a necessary condition for these ramps to exist. The implication is that the mean rate of strain acting on the turbulence somehow orients it in such a way as to cause the ramp-like large-scale features observed in the temperature signals, thus producing a non-zero derivative skewness. Tavoularis & Corrsin (1981 *a*) have recently incorporated this view in a phenomenological explanation of the observed derivative skewness in homogeneous shear flows.

A fundamental way of studying the skewness problem is to deduce the transport equation for $\overline{(\partial\theta/\partial x_1)^3}$ (and, if possible, for S itself), and examine the equation term by term. Wyngaard (1976) has in fact made such an attempt. Unfortunately, such an attempt suffers from difficulties in estimating the various terms accurately. For the present, it appears reasonable to combine judiciously this basic approach with a certain degree of empiricism based on experiment.

The purpose of this note is to examine the dependence of the magnitude and sign of the derivative skewness on the mean velocity and temperature gradients. Experiments are conducted in slightly heated homogeneous shear flows with homogeneous and non-homogeneous temperature fields, as well as in non-sheared 'isotropic' turbulence. The mean velocity and mean temperature fields in these simple flows can be prescribed independently, and thus a controlled study of the problem on hand is possible. The general conclusions drawn from these measurements are then used in exploring possible simplifications of the transport equation for $\overline{(\partial\theta/\partial x_1)^3}$. A comparison is then made of the predictions from this simplified equation with the measured derivative skewness values. Finally, the present data are used also to provide a basis for comparison with measurements in inhomogeneous shear flows.

2. Experimental set-up and instrumentation

2.1. *Description of flows*

Most measurements were made in a wind tunnel with a contraction ratio of 9 and a nominally 30×30 cm square, 330 cm long, test section. Measurements were made in the following flows:

(*a*) In the region of near-homogeneity and isotropy behind a uniformly heated grid of round rods with a square mesh of 2·54 cm and a solidity of 0·36. R_λ ($\equiv u_1'\lambda/\nu$, with the usual definition that u_1' is the r.m.s. streamwise velocity fluctuation and λ is the Taylor microscale) in this flow was about 65 at $x_1/M = 50$. All rods were uniformly heated electrically. The resulting maximum mean temperature rise of 2·5 °C was assumed to be small enough to allow temperature to be treated as a passive scalar.

(*b*) In a flow such as (*a*), but with the horizontal rods of the grid heated to different temperatures, so that a (constant or variable) mean temperature gradient was created. The maximum mean temperature rise was again about 2·5 °C.

(*c*) In three homogeneous shear flows, two of which ($dU_1/dx_2 = 13{\cdot}5\ \mathrm{s}^{-1}$ and $15{\cdot}6\ \mathrm{s}^{-1}$, x_2 being the transverse co-ordinate) were created using the shear flow generator of Rose (1966) at two different centre-line mean speeds, and the third ($dU_1/dx_2 = 44\ \mathrm{s}^{-1}$) using that of Tavoularis & Corrsin (1981*b*). In the low-shear experiments, the desired mean temperature profiles were created by heating the horizontal rods of the grid used in (*a*), located 20 mesh sizes upstream of the shear generator. This arrangement is complementary to that used by Rose (1970), and gives reasonably good linear velocity profiles. In these flows, R_λ varied (typically) between about 70 at $x_1/h = 5$ (where h is the height of the tunnel) and 130 at $x_1/h = 10$; the corresponding variation in the high-shear case was between about 100 and about 150.

For local isotropy to hold, clearly, the Reynolds and Péclet numbers must be large. An examination of table 1 (or, better still, the summary sketch of Sreenivasan & Antonia 1977) indicates that there is no discernible trend of $|S|$ with R_λ for $R_\lambda \geqslant 60$. Thus, it is conceivable that results concerning S in the present flows would also apply qualitatively to higher R_λ sheared turbulence.

(*d*) Additional measurements were also made behind a low-solidity (0·023) square-mesh heated screen (mesh size 1·11 cm) located at 20 or 34 mesh lengths behind an unheated round-rod grid of 2·54 cm square mesh and 0·44 solidity. R_λ in this flow was about 40. The essential idea of this experiment was to create a temperature field independent of the velocity field. The low screen solidity and the small screen-wire Reynolds number ($\simeq$ 38) ensured that the screen created minimum disturbance to the velocity field created by the grid (Sreenivasan *et al.* 1980). The maximum mean temperature rise was about 1 °C. These measurements were made in another wind tunnel with a 30 × 46 cm nominal test section.

2.2. *Instrumentation*

A DISA manufactured X-wire probe with wires 5 μm in diameter and 1·2 mm long was used for measuring velocity fluctuations. The hot wires were operated on two DISA 55D01 constant temperature anemometers powered by d.c. power supplies. The local mean temperature as well as the reference temperature upstream of the heating system were measured by two Fenwal Electronics GC32M21 thermistor probes. The fluctuating temperature was measured with cold wires of diameter 0·6 μm (home-made, $\simeq$ 0·8 mm long) and 1 μm (made by DISA, 0·4 mm long). Previous studies (e.g. LaRue, Deaton & Gibson 1975, Højstrup, Rasmussen & Larson 1976) have shown that the -3 dB points for fresh wires are around 5 kHz for 0·6 μm wire and 3 kHz for the DISA wires. The cold wires were operated by a home-made constant current source (Tavoularis 1978*a*). The operating current of 0·3 mA was low enough to ensure a negligible velocity sensitivity of the cold wires.

The analysis to be presented in § 4 shows the need to measure the quantity

$$\overline{(\partial u_2/\partial x_1)\,(\partial \theta/\partial x_1)^2},$$

where u_2 is the component of the fluctuating velocity in x_2 direction. For this purpose, a DISA cold-wire was positioned vertically about 0·5 mm from the nearest wire of the X-wire probe. The temperature dependence of the hot-wires in a heated flow was accounted for through the relation

$$\frac{I^2 R_w}{R_w - R(T)} = A + BU_1^n, \tag{1}$$

where A and B are calibration constants and R_w is the hot-wire resistance. The effect of temperature appears entirely through the (hypothetical) unheated wire resistance $R(T)$. The exponent n is determined so as to give the best fit to the data in the least-square sense. The adequacy of (1) for small heating has been established, for example, by Antonia *et al.* (1975) and Tavoularis (1978*b*).

All the signals were sufficiently pre-amplified before being digitized and processed on a PDP 11/40 computer. All measured quantities were corrected for noise assuming that the signal and noise were uncorrelated. The streamwise derivatives $\partial u_2/\partial x_1$ and $\partial \theta/\partial x_1$ were obtained by differencing the digitized signals and assuming that Taylor's 'frozen flow' approximation is valid.

Description of flow	dU_1/dx_2	dT/dx_2	S
(a) Grid turbulence behind a uniformly heated screen or grid	0	0	0†
(b) Grid turbulence with linear or asymmetric mean temperature gradient	0	Non-zero (+ or −)	0†
(c) Uniformly sheared flow with different mean temperature gradients	Non-zero (+ or −)	0	0†
	+	+	−‡
	+	−	+‡
	−	+	+‡
	−	−	−‡

† This value is not exactly zero, but of the order of 0·1. When corrected for the velocity sensitivity (following Wyngaard 1971 and Gibson *et al.* 1977), it is usually negligibly small in the case of the heated screen. In the heated grid case, a small residual value of S still remains (see also Antonia *et al.* 1978), possibly due to the initial coupling of the velocity and temperature fields.

‡ The magnitude itself depends on other conditions, as discussed later in this section.

TABLE 2. Qualitative summary of the results of measurements in homogeneous flows.

3. Results

Table 2 shows qualitatively the results of the present measurements. Two chief conclusions are immediate:

(*a*) The derivative skewness S is non-zero only when both $dU_1/\partial x_2$ and dT/dx_2 are non-zero. This is the most significant difference between homogeneous and inhomogeneous shear flows. In the latter category of flows such as heated wakes and jets, the skewness of the temperature derivative is non-zero even along the line of symmetry (e.g. Freymuth & Uberoi 1971, 1973; Gibson *et al.* 1977; Sreenivasan & Antonia 1977), where both the mean velocity and temperature gradients are locally zero. Because of the strong inhomogeneities and the associated large-scale transport, it is not hard to see why local conditions alone are not relevant in these flows; see also § 5.

(*b*) As a special case of a more general expression suggested by Mestayer *et al.* (1976) and Gibson *et al.* (1977), the sign of S is given by

$$-\operatorname{sgn}(dU_1/dx_2) \times \operatorname{sgn}(dT/dx_2). \qquad (2)$$

Before we can examine the dependence of the magnitude of S on dU_1/dx_2 and dT/dx_2, it is essential to have some background information about the homogeneous shear flows mentioned in (*c*) of § 2. These flows undergo an initial streamwise development, but attain a certain asymptotic state in which, while each turbulent quantity individually varies with streamwise distance, the non-dimensional ratios of turbulence quantities do not. All available experimental data on homogeneous shear flows (summarized by Sreenivasan 1979) suggest that this asymptotic state is attained when the non-dimensional parameter $(x_1/U_1)(dU_1/dx_2) \gtrsim 4$. (The similar estimate provided by Harris, Graham & Corrsin (1977) is somewhat conservative.) For the high shear case, this state is reached for $x_1/h \gtrsim 4$–5, while for the low shear cases, this state is barely reached at $x_1/h = 11$. For this reason, measurements were made

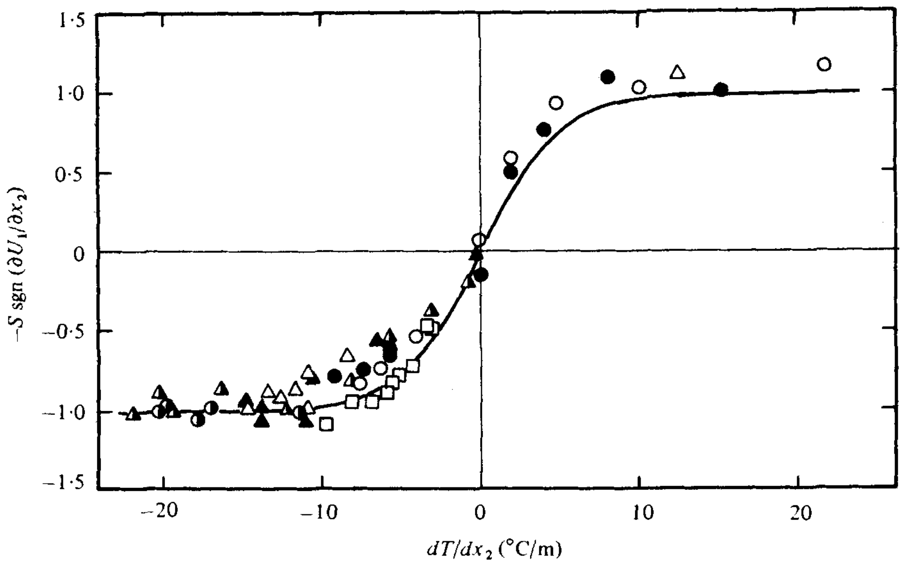

FIGURE 1. Variation of S with dT/dx_2 in the asymptotic region of homogeneous shear flow. ○, ●, ◑, $dU_1/dx_2 = 13{\cdot}5\ \mathrm{s}^{-1}$; ▲, ▲, △, $dU_1/dx_2 = 15{\cdot}6\ \mathrm{s}^{-1}$; □, $dU_1/dx_2 = 44\ \mathrm{s}^{-1}$. Different symbols in the first two sets of dU_1/dx_2 correspond to different temperature profiles. ——, tanh $(0{\cdot}20\,dT/dx_2)$, dT/dx_2 in °C m^{-1}.

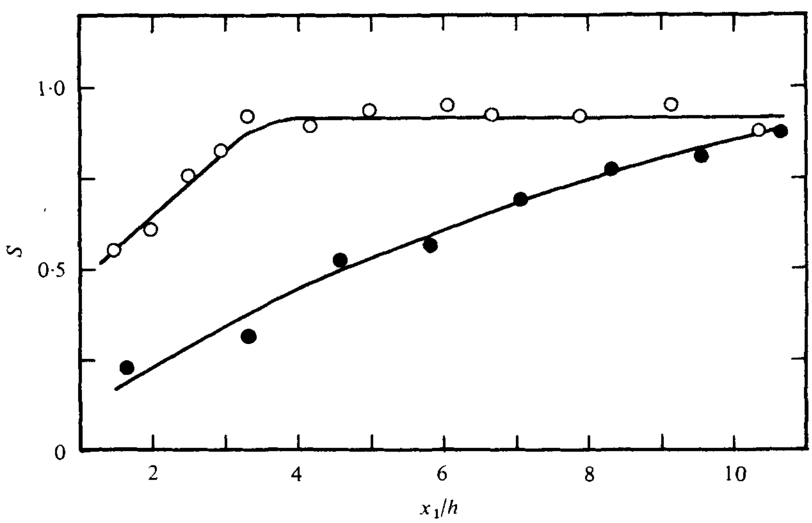

FIGURE 2. Downstream variation of S in two homogeneous shear flows. ○, $dU_1/dx_2 = 44\ \mathrm{s}^{-1}$, $dT/dx_2 = 20$ °C m^{-1}; ●, $dU_1/dx_2 = 13{\cdot}4\ \mathrm{s}^{-1}$; $dT/dx_2 = 18$ °C m^{-1}; ——, mean lines through the data.

in the region $5 \lesssim x_1/h \lesssim 11$ in the high shear experiment, and at $x_1/h = 11$ (unless otherwise specified) in the low shear cases.

Figure 1 shows the variation of S with dT/dx_2. Although the behaviour of the skewness data in figure 1 seems to depend on the precise value of the mean shear, the general trend seems reasonably independent of it. S is roughly antisymmetric with

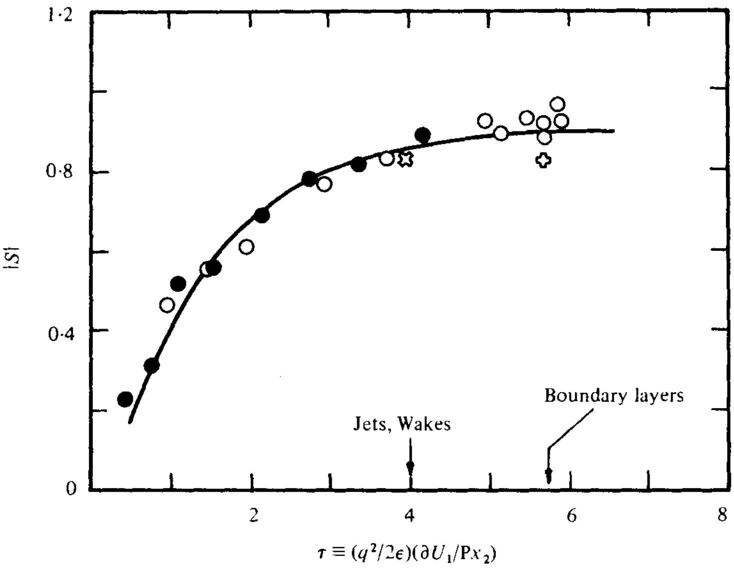

FIGURE 3. Variation of $|S|$ with the parameter τ. ○, ●, as in figure 2; ✡, average value for wakes and jets ($\tau \simeq 4$); ✡, average value for boundary layers ($\tau \simeq 5{\cdot}75$). ——, $1-1{\cdot}2\exp(-0{\cdot}77\,\tau)$.

dT/dx_2 and, for sufficiently large $|dT/dx_2|$, has essentially the same magnitude independent of its sign. This suggests that in the measuring process the velocity contamination of the temperature signal must be small, because the error due to this effect is always positive (Wyngaard 1971). The general behaviour can be idealized by a relation of the type

$$-S\,\mathrm{sgn}\,(dU_1/dx_2) = \tanh\left(\alpha\frac{dT}{dx_2}\right), \tag{3}$$

where the constant $\alpha \simeq 0{\cdot}2$ when dT/dx_2 is expressed in °C m^{-1}. Relation (3) is also shown in figure 1.

Regarding the dependence of the magnitude of S on the mean shear dU_1/dx_2, it is clear from figure 1 that the nearly constant value of S attained for sufficiently large $|dT/dx_2|$ is independent also of dU_1/dx_2. But, if the flow is still developing, $|S|$ increases with x_1 even for fixed dU_1/dx_2 and dT/dx_2. Figure 2 shows two cases with comparable dT/dx_2, but substantially different dU_1/dx_2. For the flow with larger dU_1/dx_2, $|S|$ initially increases with increasing x_1, but settles down to a constant ($\simeq 0{\cdot}9$) for $x_1/h \gtrsim 4$. But, for the flow with the lower mean shear ($dU_1/dx_2 \simeq 13{\cdot}5\ \mathrm{s}^{-1}$), $|S|$ increases monotonically with x_1, and seems to have attained the asymptotic value only around $x_1/h = 11$.

Figure 3 shows that the two sets of the skewness data coincide when plotted against the parameter $\tau \equiv (\overline{q^2}/2\epsilon)\,(dU_1/dx_2)$, where $\frac{1}{2}\overline{q^2}$ and ϵ are respectively the turbulent kinetic energy and the mean dissipation rate per unit mass; the significance of this parameter will be discussed in § 5. Experimental data can be roughly represented by the empirical relation

$$S/S^* = 1-1{\cdot}2\exp(-0{\cdot}77\tau), \tag{4}$$

where S^* is the asymptotic value of the derivative skewness for large values of τ.

It would be interesting to examine whether the skewness measurements in other (inhomogeneous) shear flows are compatible with (4). The parameter

$$\tau = (\overline{q^2}/2\epsilon)\,(\partial U_1/\partial x_2)$$

in general varies across an inhomogeneous flow such as a wake, jet or boundary layer, but there is, in all these flows, a region of nearly constant τ which can be taken to be a characteristic value. This value is approximately 4 for fully developed jets and wakes and about 5·75 for fully developed boundary layers. Figure 3 shows that all the skewness measurements in these various flows are generally consistent with (4). Because of the fairly large scatter in the measurements (see table 1), only the average value of $|S|$ is plotted here for each class of flows.

4. A simplified transport equation for $\overline{(\partial\theta/\partial x_1)^3}$

The balance equation for the instantaneous temperature fluctuation θ was derived by Corrsin (1952) as

$$\frac{\partial\theta}{\partial t}+u_j\frac{\partial T}{\partial x_j}+U_j\frac{\partial\theta}{\partial x_j}+u_j\frac{\partial\theta}{\partial x_j}-\overline{u_j\frac{\partial\theta}{\partial x_j}}=\gamma\frac{\partial^2\theta}{\partial x_j\,\partial x_j}, \tag{5}$$

where capital letters indicate mean quantities, lower-case letters indicate fluctuations, and γ is the thermal diffusivity. Differentiating (5) with respect to x_1, multiplying the resulting equation by $(\partial\theta/\partial x_1)^2$, noting that

$$\left(\frac{\partial\theta}{\partial x_1}\right)^2\frac{\partial^2\theta}{\partial x_1\,\partial t}=\tfrac{1}{3}\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial x_1}\right)^3, \tag{6a}$$

$$\left(\frac{\partial\theta}{\partial x_1}\right)^2\frac{\partial^2\theta}{\partial x_1\,\partial x_j}=\tfrac{1}{3}\frac{\partial}{\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^3, \tag{6b}$$

and, finally, averaging, we get the general transport equation for $\overline{(\partial\theta/\partial x_1)^3}$ as

$$\begin{aligned}\tfrac{1}{3}\frac{\partial}{\partial t}\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^3}&+\frac{\partial U_j}{\partial x_1}\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^2\frac{\partial\theta}{\partial x_j}}+\tfrac{1}{3}U_j\overline{\frac{\partial}{\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^3}\\&+\frac{\partial T}{\partial x_j}\overline{\frac{\partial u_j}{\partial x_1}\left(\frac{\partial\theta}{\partial x_1}\right)^2}+\frac{\partial^2 T}{\partial x_1\,\partial x_j}\overline{\left(u_j\frac{\partial\theta}{\partial x_1}\right)^2}+\overline{\frac{\partial u_j}{\partial x_1}\frac{\partial\theta}{\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^2}\\&+\overline{\tfrac{1}{3}u_j\frac{\partial}{\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^2}-\frac{\partial}{\partial x_1}\overline{\left(u_j\frac{\partial\theta}{\partial x_j}\right)}\cdot\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^2}=\gamma\overline{\frac{\partial^3\theta}{\partial x_1\,\partial x_j\,\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^2}.\end{aligned} \tag{7}$$

In a steady, 'homogeneous', shear flow with a linear temperature profile where

(i) $\dfrac{\partial}{\partial t}\overline{(\;)}=0;$

(ii) $U_2=U_3=0,\quad \dfrac{\partial U_1}{\partial x_2}\equiv\dfrac{dU_1}{dx_2}=\text{constant},$

$\dfrac{\partial T}{\partial x_1}=\dfrac{\partial T}{\partial x_3}=0,\quad \dfrac{\partial T}{\partial x_2}\equiv\dfrac{dT}{dx_2}=\text{constant},$

(iii) $\dfrac{\partial}{\partial x_1}\overline{u_1\left(\dfrac{\partial\theta}{\partial x_1}\right)^3}\ll U_1\dfrac{\partial}{\partial x_1}\overline{\left(\dfrac{\partial\theta}{\partial x_1}\right)^3},$

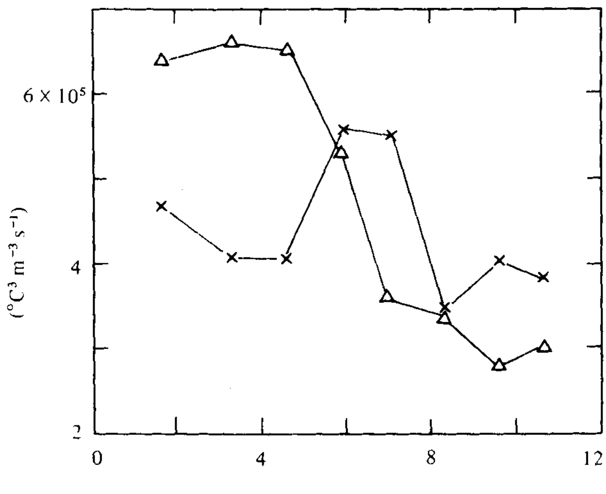

FIGURE 4. An approximate balance between the left-hand side and the first term on the right-hand side of equation (8). △, $\frac{1}{3}U_1\,(\partial/\partial x_1)\overline{(\partial\theta/\partial x_1)^3}$; ×, $-(\partial T/\partial x_2)\overline{(\partial u_2/\partial x_1)\,(\partial\theta/\partial x_1)^2}$; $dU_1/dx_2 = 13{\cdot}4\ \mathrm{s}^{-1}$.

equation (7) simplifies to

$$\tfrac{1}{3}U_1\frac{\partial}{\partial x_1}\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^3} = -\frac{\partial T}{\partial x_2}\overline{\frac{\partial u_2}{\partial x_1}\left(\frac{\partial\theta}{\partial x_1}\right)^2} - \overline{\frac{\partial u_j}{\partial x_1}\frac{\partial\theta}{\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^2} + \frac{\partial^2\overline{u_j\theta}}{\partial x_1\,\partial x_j}\cdot\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^2} + \gamma\,\overline{\frac{\partial^3\theta}{\partial x_1\,\partial x_j\,\partial x_j}\left(\frac{\partial\theta}{\partial x_1}\right)^2}. \quad (8)$$

Note that the effect of the mean shear dU_1/dx_2 is only indirectly felt since it does not appear explicitly in (8); no simple dependence of S on the mean velocity gradient can thus be expected. On the other hand, the mean temperature gradient dT/dx_2 appears as a multiplying factor in the first term on the right-hand side of (8).

In homogeneous and locally isotropic turbulence, each term in (8) is separately zero; clearly then, we are here seeking some subtle failure of the concepts of local isotropy. Since conventional order of magnitude estimates of derivatives and products of derivatives usually resort to local isotropy, such estimates for all the terms cannot in general be expected to be meaningful, although it is possible that some terms may behave strictly as required by local isotropy. Such order of magnitude estimates are especially difficult for the second and fourth terms on the right-hand side of (8); see Wyngaard (1976).

Therefore, it appears that further rational simplification of (8), based on order-of-magnitude estimates alone, is not possible at this stage. We shall now make use of an observation reported in § 3, namely that if we let dT/dx_2 become zero (by heating the flow to a uniform mean temperature), all other conditions remaining the same, the skewness of $\partial\theta/\partial x_1$ is zero. This means that under those conditions there is a balance between the second, third, and fourth terms on the right-hand side of (8). There is no good *a priori* reason to believe that these same three terms add up to zero when dT/dx_2 is non-zero, but measurements seem to support such a conjecture. Figure 4

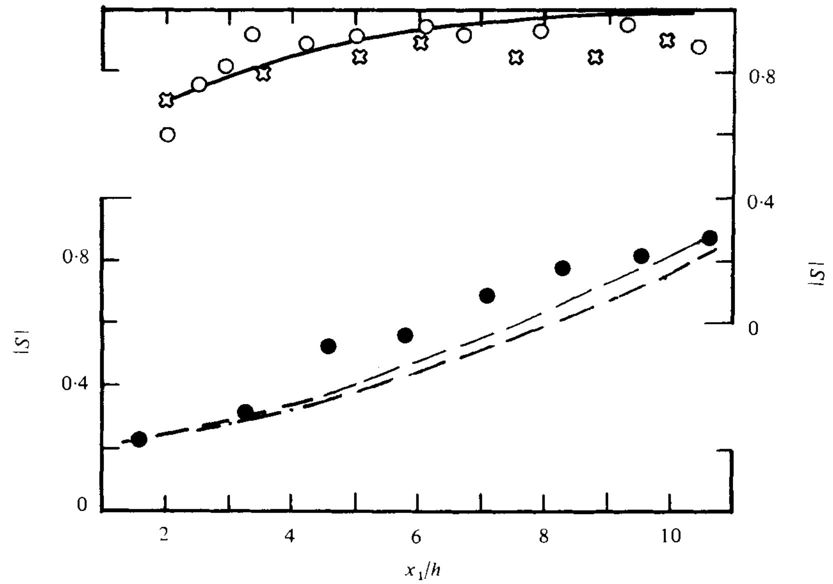

FIGURE 5. Comparison of the measured derivative skewness with the values evaluated according to (10). ○, $dU_1/dx_2 = 44\ \mathrm{s}^{-1}$, $dT/dx_2 = 20\ °\mathrm{C\ m}^{-1}$; × are repeat measurements. ——, calculation according to (10), with $\xi/h = 2$. ●, $dU_1/dx_2 = 13{\cdot}4\ \mathrm{s}^{-1}$, $dT/dx_2 = 18-20\ °\mathrm{C\ m}^{-1}$; – – –, calculation according to (10) with $dT/dx_2 = 20\ °\mathrm{C\ m}^{-1}$, — · —, calculation with $dT/dx_2 = 18\ °\mathrm{C\ m}^{-1}$. $\xi/h = 1{\cdot}5$.

shows the measured values of the left-hand side and of the first term on the right-hand side of (8). Considering the uncertainties in the graphical differentiation of the measured $\overline{(\partial\theta/\partial x_1)^3}$ and in the measurement of $\overline{(\partial u_2/\partial x_1)(\partial\theta/\partial x_1)^2}$, it appears that a rough balance prevails between the two terms; the balance for the high shear case (not shown in the figure) is even better. The only other term we have been able to measure with any reliability, namely the third term on the right-hand side of (8), was found to be very small in comparison.

Based on this evidence, we suggest that an approximate balance of the type

$$\tfrac{1}{3}U_1\frac{\partial}{\partial x_1}\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^3} \approx -\frac{dT}{\partial x_2}\overline{\frac{\partial u_2}{\partial x_1}\left(\frac{\partial\theta}{\partial x_1}\right)^2} \qquad (9)\dagger$$

governs the streamwise development of $\overline{(\partial\theta/\partial x_1)^3}$, at least for the flows examined here. This approximation does not imply anything *individually* for the neglected terms, mainly the second and fourth terms on the right hand side of (8), but it suggests that their sum essentially vanishes. Some further justification for (9) can be obtained from a consideration of the linearized equation for θ, from which an explicit expression for

† Here, the right-hand side of (9) is independent of x_2 by transverse homogeneity, whereas the left-hand side depends on x_2 through U_1. Inconsistency of this kind has previously been noted for second-order correlations (Champagne, Harris & Corrsin 1970; Harris *et al.* 1977; Tavoularis & Corrsin 1981*b*), and arises essentially because steady homogeneous flow with mean shear is impossible to obtain. In a moving frame, of course, the operator $U_1(\partial/\partial x_1)$ should be interpreted as d/dt.

$\overline{(\partial\theta/\partial x_1)^3}$ can be obtained. This expression shows that $\overline{(\partial\theta/\partial x_1)^3}$ depends directly on $\partial T/\partial x_2$ and on statistical quantities involving $\partial u_2/\partial x_1$, somewhat as in (9).

Since our concern is the skewness of $\partial\theta/\partial x_1$, we now integrate (9) along x_1 (for a given x_2/h) to obtain

$$\overline{\left(\frac{\partial\theta}{\partial x_1}\right)^3_{\text{at } x_1}} = \overline{\left(\frac{\partial\theta}{\partial x_1}\right)^3_{\text{at } \xi}} - \frac{3}{U_1}\int_{\xi}^{x_1} \frac{dT}{\partial x_2} \overline{\frac{\partial u_2(x')}{\partial x_1'}\left(\frac{\partial\theta(x')}{\partial x_1'}\right)^2} dx_1'. \tag{10}$$

The integrand being a measured quantity, we can obtain $\overline{(\partial\theta/\partial x_1)^3}$ at any x_1 knowing its value at an initial station $x_1 = \xi$. This calculated $\overline{(\partial\theta/\partial x_1)^3}$ can now be divided by the measured $\overline{(\partial\theta/\partial x_1)^2}^{\frac{3}{2}}$ at that x_1 to obtain the calculated skewness. Figure 5 shows that the calculated and the measured values of S are of comparable magnitudes. It is interesting to note that equation (10) predicts an S that hardly varies with x_1 in the high shear experiment, while also predicting that S varies substantially with x_1 in the low shear case, just exactly as the measurement shows. Figure 5 also shows that the choice of the origin ξ is not crucial to this conclusion.

5. Discussion and conclusions

The observation in § 3 that in homogeneous flows the skewness S of the temperature derivative is non-zero only when dU_1/dx_2 and dT/dx_2 are both non-zero considerably simplifies the transport equation for the third moment of the temperature derivative. According to the simplified equation (9), $\overline{(\partial\theta/\partial x_1)^3}$ depends essentially on the term $(dT/dx_2)\,\overline{(\partial u_2/\partial x_1)\,(\partial\theta/\partial x_1)^2}$. This term obviously vanishes with dT/dx_2; it also vanishes when the mean velocity is uniform (because of reflexional symmetry).

The sign of S is determined by those of dU_1/dx_2 and dT/dx_2 according to (2). The magnitude of S bears a fairly straightforward relation to that of dT/dx_2 according to (3): for small dT/dx_2, $S \propto dT/dx_2$, but is independent of dT/dx_2 for large $|dT/dx_2|$. However, the dependence of S on dU_1/dx_2 is more involved. It appears from figure 3 that the parameter τ is reasonably successful in correlating the skewness data for a fairly wide variety of flow situations. One possible interpretation of τ is that it is a product of the rate of strain and a characteristic time over which this rate of strain acts on an identifiable entity of turbulence. For the energy containing eddies, the characteristic time is $\overline{q^2}/2\epsilon$, and τ can be interpreted as the total strain† that acts on the energy containing eddies during their life time.

The seemingly unique dependence of $|S|$ on total strain (instead of the rate of strain) reminds us of the rapid-distortion situation (e.g. Ribner & Tucker 1953; Batchelor & Proudman 1954; Townsend 1976). Although the plane shear strain imposed by the mean velocity distribution in shear flows (including the case of homogeneous shear flows studied here) is not 'large' in the rapid-distortion sense, Townsend's (1976)

† The interpretation that τ is a total strain is not altogether unequivocal, because it can also be interpreted (perhaps somewhat less convincingly) as a dimensionless mean strain rate $(dU_1/dx_2)/(u'/l)$, where u'/l is a large eddy turn-over rate. Noting that $\epsilon \sim u'^3/l$, we have

$$\frac{dU_1/dx_2}{u'/l} \sim \frac{u'^2}{\epsilon}\frac{dU_1}{dx_2} \sim \tau.$$

We will not consider this further.

calculations show that the turbulence *structure* observed in these shear flows is essentially the same as that produced by the application of a 'large' shear strain rate for a 'small' amount of time on an initially isotropic (or 'structureless') field, as long as the total strain is the same in both cases.

The definite relation observed between $|S|$ on one hand and dU_1/dx_2 and dT/dx_2 on the other suggests the following possible physical mechanism. The straining effect of the mean shear causes a preferential orientation of the large structure; in the presence of a mean temperature gradient, the heat transport due to these preferentially oriented structures results in ramp-like character in the temperature signal, thus resulting in non-zero skewness value of the derivative.† The larger the mean shear the quicker (in terms of the large eddy turn-over time) is the asymptotic value of S reached. The fact that this asymptotic value (attained for 'large' τ, empirically for $\tau \gtrsim 4$) is independent of dU_1/dx_2 and dT/dx_2 (provided the latter is greater than a certain value, see figure 1) suggests that although the straining effect due to the mean shear itself persists indefinitely, the orientation of the turbulence structure does not evolve beyond a certain 'equilibrium' state. Further orienting effect of the mean rate of strain is counteracted by some strain-relieving mechanism, so that further evolution does not occur.

As already pointed out, the chief difference in the behaviour of S between the homogeneous flows and flows such as jets, wakes, and boundary layers is that in the latter class of flows there is a large-scale inhomogeneity. Clearly, one of the chief conclusions of the present study that both dU_1/dx_2 and dT/dx_2 must be locally non-zero for S to be non-zero does not hold for the inhomogeneous flows, as observations of S on the centre-line of jets and wakes‡ have shown. Another situation where this rule may not work is in the presence of a thermal interface in a homogeneous turbulent flow. Perhaps for inhomogeneous flows a suitable global average of $\partial U_1/\partial x_2$ and $\partial T/\partial x_2$ should replace the local values of the present analysis. The physical mechanism need not, however, be very different, but only need be modified by allowing for the bulk transport.

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† The ramp model also successfully explains the measured skewness of temperature derivatives in x_2 and x_3 directions (Sreenivasan, Antonia & Danh 1977).

‡ In the traditional experiments on heated (or cooled) wakes and jets, the maxima (or the minima) of mean velocity and mean temperature profiles coincide. Consequently for a more crucial test of some of the hypotheses made here, especially designed experiments have been made; these will be reported separately.

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